Estimation of moment magnitude and stress parameter from ShakeMap ground-motion parameters for real-time applications

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Abstract

We describe a method to determine moment magnitude and stress parameter in near real-time from ShakeMap ground motion parameters (5%-damped pseudo spectral acceleration [PSA] at 1 Hz, peak ground acceleration [PGA] or PSA at 10 Hz), suitable for regions having a sparse network, in the immediate aftermath of a small to moderate earthquake. The methodology is based on relating ShakeMap parameters to source and attenuation parameters within the context of a stochastic point-source model, in order to provide an event-specific ground-motion prediction equation (GMPE) that will reliably predict amplitudes across the region. An example application is provided for southern Ontario. Here, we initially develop a simulation-guided regional generic GMPE model based on the available database. Then, using the estimated regional attenuation and source parameters, we are able to invert ShakeMaps parameters to estimate moment magnitude and stress parameter in near real-time, in order to provide reliable event-specific GMPEs. The event-specific GMPE are then used to provide robust, calibrated ShakeMaps that are fully consistent with ground-motion observations.

Intoroduction

Real-time ShakeMaps, showing the intensity of shaking over the region and/or at specific sites of interest, have become a valuable tool for real-time hazard and risk management worldwide. Because of the popularity of these tools, calculation of ShakeMap ground motion parameters has become routine on many standard seismological platforms. The ShakeMap parameters include peak ground acceleration (PGA) and velocity (PGV), and the 5% damped pseudo-spectral acceleration response (PSA) at selected frequencies, typically 0.33 Hz, 1 Hz, 3.33 Hz, and 10 Hz. For many applications such as real-time ShakeMaps and the development of ground-motion prediction equations (GMPEs), it is useful to have estimates of moment magnitude (**M**) and stress parameter ($\Delta\sigma$). This two-parameter representation provides a fairly-comprehensive summary of the source parameters of the event, in the context of the simple Brune point-source model [*Brune* 1970, also referred to as the ω^2 model], in which **M** controls near-source spectral amplitudes at low frequencies, while $\Delta\sigma$ controls those amplitudes at high frequencies.

In many regions of low to moderate seismicity, such as eastern North America (ENA), ground-motion estimates are required for the small to moderate events that happen several times per year. Such events may not be very damaging but rapid information on the amplitudes of shaking is required to inform the public, and government or regulatory officials as appropriate.

In this paper, we lay out a methodology to determine **M** and $\Delta\sigma$ from ShakeMap ground-motion parameters (PSA at 1 Hz or 0.33 Hz, PGA or PSA at 10 Hz). The methodology is suitable for use in the immediate aftermath of a small to moderate earthquake (**M**<6). An example of this approach provided for southern Ontario. We first develop a regionally adjusted generic GMPE [*Yenier and Atkinson*, 2015] using the observed data in southern Ontario. Then, by implementing the derived source and attenuation parameters, we establish a simple approach for inverting the ShakeMaps parameters to estimate **M** and $\Delta\sigma$ in near real-time.

Methodology

The essential ground-motion information that is used to produce real-time maps of earthquake shaking are selected ShakeMap parameters. Here we focus on the use of the 1 Hz and 10 Hz PSA, though we also discuss alternative formulations using the 0.33 Hz PSA (useful if the calculated **M**>6) or the PGA (useful if the 10 Hz PSA is unavailable

To estimate **M** and $\Delta\sigma$ from the observed PSA values, we relate these source parameters to the predictions of an equivalent point-source stochastic model that has been optimized for the attenuation and site response attributes of the region, as described by [*Yenier and Atkinson 2015*a,b]. The generic GMPE, formulated from predictions of the stochastic point-source simulations, is parameterized so as to isolate the effects of the basic source and attenuation parameters on peak ground motions and response spectra. In the formulation of the generic GMPE, the source and the geometrical spreading functions are obtained using the simulated ground-motion parameters form the reference model developed by Yenier and Atkinson [2015a], while the anelastic attenuation term and the overall calibration factor can be obtained using available empirical data. We define the generic GMPE formulation as follow:

$$\ln Y = F_E + F_Z + F_\gamma + F_S + C \tag{1}$$

where $\ln Y$ is the natural logarithm of a ground-motion intensity measure, such as the PSA at a selected frequency. F_E, F_Z, F_γ and F_S represent functions for source, stress adjustment, geometrical spreading, anelastic attenuation and site effects, respectively. The C term is an empirical calibration factor that accounts for the residual differences between simulations and empirical data. The source function (F_E) describes the effects of magnitude and stress parameter on ground-motion amplitudes as:

$$F_E = F_M + F_{\Delta\sigma} \tag{2}$$

where F_M is the magnitude scaling function and assumed to be the same as the reference model magnitude scaling function, which was obtained for $\Delta\sigma$=100 bars and near-surface attenuation parameter (κ_0) equal to 0.025. $F_{\Delta\sigma}$ is the stress adjustment factor that is needed when $\Delta\sigma$ is different than 100 bars. The geometrical spreading function (F_Z) is also assumed to be the same as the reference model geometrical spreading function, which is a bilinear function with transition distance at 50 km. The anelastic attenuation function is also defined as:

$$F_\gamma = \gamma D_{rup} \tag{3}$$

where γ is a frequency-dependent anelastic attenuation coefficient which can be fined using the observed database, and D_{rup} is the closest distance from the site to the fault-rupture surface. The site effect function (F_S) is the relative site amplification to the reference site condition, which is NEHRP (National Earthquake Hazards Reduction Program) B/C site condition or weighted average shear-wave velocity over the top 30 m is V_{S30} ~ 760m/sec. A number of approaches could be used to determine F_S, such as regression of the observed data or using and empirical site amplification function.

We can readily invert the generic GMPE as described in the foregoing to estimate **M** and $\Delta\sigma$ from the ground-motion observations of an individual event. Note that we require knowledge of the event location so that the distance to each station can be estimated. The procedure to obtain **M** and $\Delta\sigma$ for each event is as follow: 1) Remove the estimated site amplification form the observed ground-motion parameters, 2) Use 1 Hz PSA data to obtain an estimate of **M** from each observation and find the average value for all of the stations. For small to moderate magnitude events, 1 Hz data is not sensitive to $\Delta\sigma$ and a regional $\Delta\sigma$ value can be used to estimate **M**. This can be done either by inverting Eq. (1) or by using a grid search to find the **M** which minimizes the residuals at 1 Hz. It should be noted that for larger magnitude events one can use 0.33 Hz data as 1 Hz data become sensitive to $\Delta\sigma$, 3) Use 10 Hz PSA data and the estimated **M** value from step 2 to find the value of $\Delta\sigma$ that minimizes the residuals over all of the stations. If 10 Hz PSA is not available, PGA can also be used to estimate $\Delta\sigma$.

Application to Southern Ontario

The Southern Ontario Seismic Network (SOSN) is operated by the University of Western Ontario (Department of Earth Sciences) for Ontario Power Generation and Bruce Power. The network consists of 25 broad-band three component seismic stations shown on Figure 1, which work in conjunction with other seismic stations in the region. The database used in this study consists of 1205 horizontal ground-motion parameters (PGA, PGV and PSA at 0.5 to 20 Hz) from 62 events with magnitude larger than 2.5, recorded at up to 84 seismic stations. Figure 1 also shows the observed events in the region, and the magnitude-distance distribution of the database.

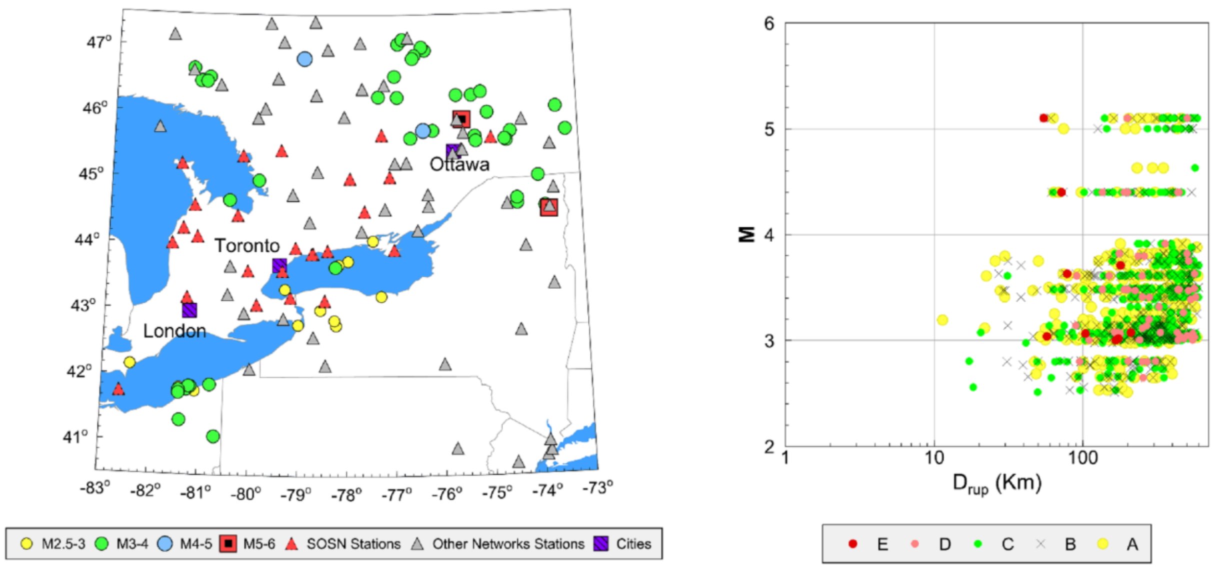

Figure 1: Left: Geographic distribution of study events and stations, Right: Magnitude-distance distribution of the database, by NEHRP site classes.

In order to find a regionally-adjusted GMPE for SOSN database, first we find the residuals by assuming the magnitude scaling and geometrical spreading functions from the reference model of Yenier and Atkinson [2015b]:

$$re_{ij} = lnY_{ij} - \left(F_{M,i} + F_{Z,ij}\right) = E_{\mathrm{i}} + \gamma_{SOSN} D_{rup,ij} + F_{S,j} \tag{4}$$

where re_{ij} and Y_{ij} are the residual and the observed horizontal ground-motion parameter for event i at station j, respectively. $F_{M,i}$, $F_{Z,ij}$, are the magnitude scaling function for event i, and the geometrical spreading function for event i and station j, respectively. E_i is the source-term for event i, which includes both the event specific stress parameter adjustment factor ($F_{\Delta\sigma,i}$) and also the calibration factor (C). The regional anelastic attenuation is given by γ_{SOSN}, while $D_{rup,ij}$ is the closest distance to the rupture surface of event i from station j. $F_{S,j}$ is the site effect term for station j relative to the assumed reference site condition in the reference model (B/C site condition).

For solving Eq. (4), we follow the generalized inversion scheme introduced by [Andrews 1986] to solve for the event term (E_i), the regional anelastic attenuation term (γ_{SOSN}) and the site term ($F_{S,j}$). In order to remove the trade-off between the source term and the site term, we need to assume a reference site condition with known site amplification. Here we assume that the reference site condition is the very hard rock site condition that is typical of seismograph sites in eastern Canada; this corresponds to sites with V_{S30} ~ 2000 m/sec. We assign zero site amplification to the assumed reference site condition, averaged across all such stations; therefore, the GMPE model will be calibrated for an average site condition of hard rock.

The first output of the generalized inversion is the site amplification term (F_S) relative to the assumed reference site condition (V_{S30} ~ 2000 m/sec) for each of the individual stations. The determined site amplifications can be used in the first step of **M** and $\Delta\sigma$ estimation to remove the site effects and level all of the stations to the reference site condition. The second output is the regional anelastic attenuation term (γ_{SOSN}) which is shown in Figure 2. As we observe here, the anelastic attenuation term in southern Ontario indicates slower attenuation in comparison to the anelastic attenuation obtained for the broader region of Central and eastern North America as a whole [CENA; *Yenier and Atkinson*, 2015b].

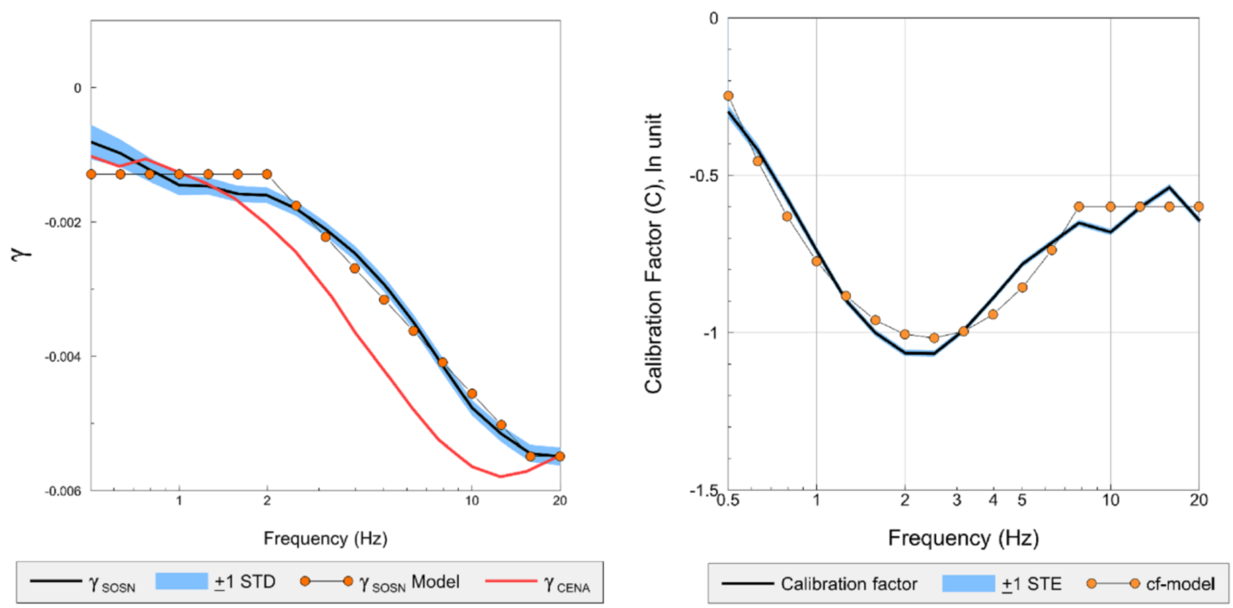

Figure 2: Left: Derived anelastic attenuation as a function of frequency, Right: Derived calibration factor as a function of frequency. Blue band indicates one standard deviation (STD) or one standard error (STE) about the determined values for southern Ontario.

The next step is to find the event-specific stress parameter ($\Delta\sigma_i$) and the regional calibration factor (C) using the derived source term (E_i). We find the event-specific stress parameter by matching the shape of the source term [Yenier and Atkinson, 2015a,b] for each event. We generalize the resulting values as a stress parameter model ($\Delta\sigma_{SOSN}$), which is a function of depth (d) and magnitude ($\boldsymbol{M}$):

$$ln\Delta\sigma_{SOSN} = 6.10 + \min[0,0.37(d-7.5)] + \min[0,1.12(\boldsymbol{M}-3.5)] \quad (5)$$

The average regional calibration factor (C_{SOSN}) can then be found by removing the stress parameter adjustment factor ($F_{\Delta\sigma}$) from the source terms (E_i). The regional calibration factor term is shown in Figure 2. This factor compensates for any difference that we might expect between the observed ground-motion parameters and the simulated ground-motion parameters. The final formulation of the regionally-adjusted GMPE can be written as follows, in which Y is the predicted ground-motion parameter:

$$\ln(Y) = F_M + F_Z + F_{\Delta\sigma,\,SOSN} + \gamma_{SOSN} D_{rup} + F_S + C_{SOSN} \quad (6)$$

After developing the regionally-adjusted GMPE for southern Ontario, the next step is to implement the derived equation for real-time **M** and Δσ estimation using just the SOSN stations. Our approach is to produce a large set of predicted ground-motion parameters based on alternative values for the model parameters, and then use a grid search to estimate **M** and Δσ from the ShakeMaps parameters recorded for the event. We produce the expected ground-motion values for 2.5≤ **M** ≤6 in 0.1 **M** increments for 50 equally log-spaced distances from 1 km $\leq D_{rup} \leq$600 km, and for 30 equally log-spaced stress parameters from 10 bars ≤ Δσ ≤1000 bars. In the first step, we remove from each recording the site amplification terms (F_S) as obtained from the generalized inversion results. Then using the observed and predicted PSA data at 1 Hz (for the reference site condition), we find a magnitude value ($\mathbf{M}_{SOSN}$) which minimize the residuals at 1 Hz, assuming the regional stress parameter model (Eq .(5)). Then we use the observed and predicted PSA data at 10 Hz and find an event-specific stress parameter value ($\Delta\sigma_{SOSN}$) to minimize the residuals at 10 Hz, assuming the **M** value obtained from the 1 Hz PSA. We can iterate these steps one or two more times in order to obtain more precise results. Figure 3 shows an example of this approach for the 2010-06-23 Val-des-Bois earthquake (**M**5.1). As we observe here, we are able to produce an event-specific GMPE, which can closely match the observed ground-motion parameters on average over a wide frequency range. Figure 3 also shows the comparison between the estimated magnitude and the known moment magnitude, for all study events; there is a close agreement between the estimated and the known values of **M**.

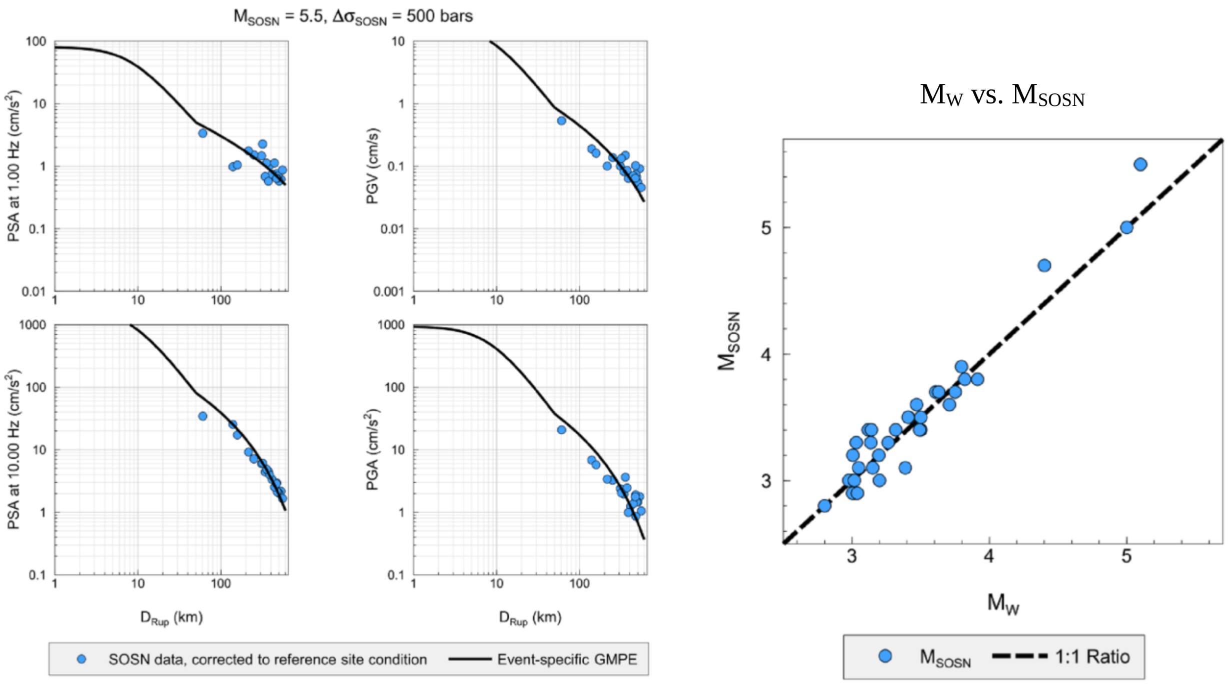

Figure 3: Left: Event-specific GMPE for Val-des-Bois earthquake (**M**5.1), Right: Comparison between the estimated **M** using the techniques of this study to the corresponding known catalogue values of **M**.

Conclusion

We establish a technique for real-time estimation of **M** and $\Delta\sigma$ using the generic GMPE approach in conjunction with recorded ShakeMaps parameters, and apply it to southern Ontario. The approach produces event-specific GMPEs which are in close agreement with the observed ground-motion data on average. The event-specific GMPE can be used as an input for ShakeMaps or any other real-time hazard and risk assessment software to provide robust, calibrated estimates of ground shaking intensity throughout the region.

References

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